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Randomness Requirements for Security

Status of This Memo

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Abstract

Security systems are built on strong cryptographic algorithms that foil pattern analysis attempts. However, the security of these systems is dependent on generating secret quantities for passwords, cryptographic keys, and similar quantities. The use of pseudo-random processes to generate secret quantities can result in pseudo-security. A sophisticated attacker may find it easier to reproduce the environment that produced the secret quantities and to search the resulting small set of possibilities than to locate the quantities in the whole of the potential number space.

Choosing random quantities to foil a resourceful and motivated adversary is surprisingly difficult. This document points out many pitfalls in using poor entropy sources or traditional pseudo-random number generation techniques for generating such quantities. It recommends the use of truly random hardware techniques and shows that the existing hardware on many systems can be used for this purpose. It provides suggestions to ameliorate the problem when a hardware solution is not available, and it gives examples of how large such quantities need to be for some applications.

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1. Introduction and Overview

Software cryptography is coming into wider use, although there is a long way to go until it becomes pervasive. Systems such as SSH, IPSEC, TLS, S/MIME, PGP, DNSSEC, and Kerberos are maturing and becoming a part of the network landscape [SSH] [IPSEC] [TLS] [S/MIME] [MAIL_PGP*] [DNSSEC*]. For comparison, when the previous version of this document [RFC1750] was issued in 1994, the only Internet cryptographic security specification in the IETF was the Privacy Enhanced Mail protocol [MAIL_PEM*].

These systems provide substantial protection against snooping and spoofing. However, there is a potential flaw. At the heart of all cryptographic systems is the generation of secret, unguessable (i.e., random) numbers.

The lack of generally available facilities for generating such random numbers (that is, the lack of general availability of truly unpredictable sources) forms an open wound in the design of cryptographic software. For the software developer who wants to build a key or password generation procedure that runs on a wide range of hardware, this is a very real problem.

Note that the requirement is for data that an adversary has a very low probability of guessing or determining. This can easily fail if pseudo-random data is used that meets only traditional statistical tests for randomness, or that is based on limited-range sources such as clocks. Sometimes such pseudo-random quantities can be guessed by an adversary searching through an embarrassingly small space of possibilities.

This Best Current Practice document describes techniques for producing random quantities that will be resistant to attack. It recommends that future systems include hardware random number generation or provide access to existing hardware that can be used for this purpose. It suggests methods for use if such hardware is not available, and it gives some estimates of the number of random bits required for sample applications.

2. General Requirements

Today, a commonly encountered randomness requirement is to pick a user password, usually a simple character string. Obviously, a password that can be guessed does not provide security. For re-usable passwords, it is desirable that users be able to remember the password. This may make it advisable to use pronounceable character strings or phrases composed of ordinary words. But this affects only the format of the password information, not the requirement that the password be very hard to guess.

Many other requirements come from the cryptographic arena. Cryptographic techniques can be used to provide a variety of services, including confidentiality and authentication. Such services are based on quantities, traditionally called "keys", that are unknown to and unguessable by an adversary.

There are even TCP/IP protocol uses for randomness in picking initial sequence numbers [RFC1948].

Generally speaking, the above examples also illustrate two different types of random quantities that may be wanted. In the case of human-usable passwords, the only important characteristic is that they be unguessable. It is not important that they may be composed of ASCII characters, so the top bit of every byte is zero, for example. On the other hand, for fixed length keys and the like, one normally wants quantities that appear to be truly random, that is, quantities whose bits will pass statistical randomness tests.

In some cases, such as the use of symmetric encryption with the one-time pads or an algorithm like the US Advanced Encryption Standard [AES], the parties who wish to communicate confidentially and/or with authentication must all know the same secret key. In other cases, where asymmetric or "public key" cryptographic techniques are used, keys come in pairs. One key of the pair is private and must be kept secret by one party; the other is public and can be published to the world. It is computationally infeasible to determine the private key from the public key, and knowledge of the public key is of no help to an adversary [ASYMMETRIC]. See general references [SCHNEIER, FERGUSON, KAUFMAN].

The frequency and volume of the requirement for random quantities differs greatly for different cryptographic systems. With pure RSA, random quantities are required only when a new key pair is generated; thereafter, any number of messages can be signed without a further need for randomness. The public key Digital Signature Algorithm devised by the US National Institute of Standards and Technology (NIST) requires good random numbers for each signature [DSS]. And

encrypting with a one-time pad (in principle the strongest possible encryption technique) requires randomness of equal volume to all the messages to be processed. See general references [SCHNEIER, FERGUSON, KAUFMAN].

In most of these cases, an adversary can try to determine the "secret" key by trial and error. This is possible as long as the key is enough smaller than the message that the correct key can be uniquely identified. The probability of an adversary succeeding at this must be made acceptably low, depending on the particular application. The size of the space the adversary must search is related to the amount of key "information" present, in an information-theoretic sense [SHANNON]. This depends on the number of different secret values possible and the probability of each value, as follows:

$$\text{Bits of information} = \sum_{i} - p_i * \log_2 (p_i)$$

where i counts from 1 to the number of possible secret values and p sub i is the probability of the value numbered i. (Because p sub i is less than one, the log will be negative, so each term in the sum will be non-negative.)

If there are 2^n different values of equal probability, then n bits of information are present and an adversary would have to try, on the average, half of the values, or 2^(n-1), before guessing the secret quantity. If the probability of different values is unequal, then there is less information present, and fewer guesses will, on average, be required by an adversary. In particular, any values that an adversary can know to be impossible or of low probability can be initially ignored by the adversary, who will search through the more probable values first.

For example, consider a cryptographic system that uses 128-bit keys. If these keys are derived using a fixed pseudo-random number generator that is seeded with an 8-bit seed, then an adversary needs to search through only 256 keys (by running the pseudo-random number generator with every possible seed), not 2^128 keys as may at first appear to be the case. Only 8 bits of "information" are in these 128-bit keys.

While the above analysis is correct on average, it can be misleading in some cases for cryptographic analysis where what is really important is the work factor for an adversary. For example, assume that there is a pseudo-random number generator generating 128-bit keys, as in the previous paragraph, but that it generates zero half of the time and a random selection from the remaining 2^128 - 1 values the rest of the time. The Shannon equation above says that there are 64 bits of information in one of these key values, but an adversary, simply by trying the value zero, can break the security of half of the uses, albeit a random half. Thus, for cryptographic purposes, it is also useful to look at other measures, such as min-entropy, defined as

$$\text{Min-entropy} = -\log\left(\max_i\left(p_i\right)\right)$$

where i is as above. Using this equation, we get 1 bit of min-entropy for our new hypothetical distribution, as opposed to 64 bits of classical Shannon entropy.

A continuous spectrum of entropies, sometimes called Renyi entropy, has been defined, specified by the parameter r. Here r = 1 is Shannon entropy and r = infinity is min-entropy. When r = zero, it is just log (n), where n is the number of non-zero probabilities. Renyi entropy is a non-increasing function of r, so min-entropy is always the most conservative measure of entropy and usually the best to use for cryptographic evaluation [LUBY].

Statistically tested randomness in the traditional sense is NOT the same as the unpredictability required for security use.

For example, the use of a widely available constant sequence, such as the random table from the CRC Standard Mathematical Tables, is very weak against an adversary. An adversary who learns of or guesses it can easily break all security, future and past, based on the sequence [CRC]. As another example, using AES with a constant key to encrypt successive integers such as 1, 2, 3, ... will produce output that also has excellent statistical randomness properties but is predictable. On the other hand, taking successive rolls of a six-sided die and encoding the resulting values in ASCII would produce statistically poor output with a substantial unpredictable component. So note that passing or failing statistical tests doesn’t reveal whether something is unpredictable or predictable.

3. Entropy Sources

Entropy sources tend to be very implementation dependent. Once one has gathered sufficient entropy, it can be used as the seed to produce the required amount of cryptographically strong pseudo-randomness, as described in Sections 6 and 7, after being de-skewed or mixed as necessary, as described in Sections 4 and 5.

Is there any hope for true, strong, portable randomness in the future? There might be. All that's needed is a physical source of unpredictable numbers.

Thermal noise (sometimes called Johnson noise in integrated circuits) or a radioactive decay source and a fast, free-running oscillator would do the trick directly [GIFFORD]. This is a trivial amount of hardware, and it could easily be included as a standard part of a computer system's architecture. Most audio (or video) input devices are usable [TURBID]. Furthermore, any system with a spinning disk or ring oscillator and a stable (crystal) time source or the like has an adequate source of randomness ([DAVIS] and Section 3.3). All that's needed is the common perception among computer vendors that this small additional hardware and the software to access it is necessary and useful.

ANSI X9 is currently developing a standard that includes a part devoted to entropy sources. See Part 2 of [X9.82].

3.1. Volume Required

How much unpredictability is needed? Is it possible to quantify the requirement in terms of, say, number of random bits per second?

The answer is that not very much is needed. For AES, the key can be 128 bits, and, as we show in an example in Section 8, even the highest security system is unlikely to require strong keying material of much over 200 bits. If a series of keys is needed, they can be generated from a strong random seed (starting value) using a cryptographically strong sequence, as explained in Section 6.2. A few hundred random bits generated at start-up or once a day is enough if such techniques are used. Even if the random bits are generated as slowly as one per second and it is not possible to overlap the generation process, it should be tolerable in most high-security applications to wait 200 seconds occasionally.

These numbers are trivial to achieve. It could be achieved by a person repeatedly tossing a coin, and almost any hardware based process is likely to be much faster.

3.2. Existing Hardware Can Be Used For Randomness

As described below, many computers come with hardware that can, with care, be used to generate truly random quantities.

3.2.1. Using Existing Sound/Video Input

Many computers are built with inputs that digitize some real-world analog source, such as sound from a microphone or video input from a camera. The "input" from a sound digitizer with no source plugged in or from a camera with the lens cap on is essentially thermal noise. If the system has enough gain to detect anything, such input can provide reasonably high quality random bits. This method is extremely dependent on the hardware implementation.

For example, on some UNIX-based systems, one can read from the /dev/audio device with nothing plugged into the microphone jack or with the microphone receiving only low level background noise. Such data is essentially random noise, although it should not be trusted without some checking, in case of hardware failure, and it will have to be de-skewed.

Combining this approach with compression to de-skew (see Section 4), one can generate a huge amount of medium-quality random data with the UNIX-style command line:

```
cat /dev/audio | compress - >random-bits-file
```

A detailed examination of this type of randomness source appears in [TURBID].

3.2.2. Using Existing Disk Drives

Disk drives have small random fluctuations in their rotational speed due to chaotic air turbulence [DAVIS, Jakobsson]. The addition of low-level disk seek-time instrumentation produces a series of measurements that contain this randomness. Such data is usually highly correlated, so significant processing is needed, as described in Section 5.2 below. Nevertheless, experimentation a decade ago showed that, with such processing, even slow disk drives on the slower computers of that day could easily produce 100 bits a minute or more of excellent random data.

Every increase in processor speed, which increases the resolution with which disk motion can be timed or increases the rate of disk seeks, increases the rate of random bit generation possible with this technique. At the time of this paper and with modern hardware, a more typical rate of random bit production would be in excess of

10,000 bits a second. This technique is used in random number generators included in many operating system libraries.

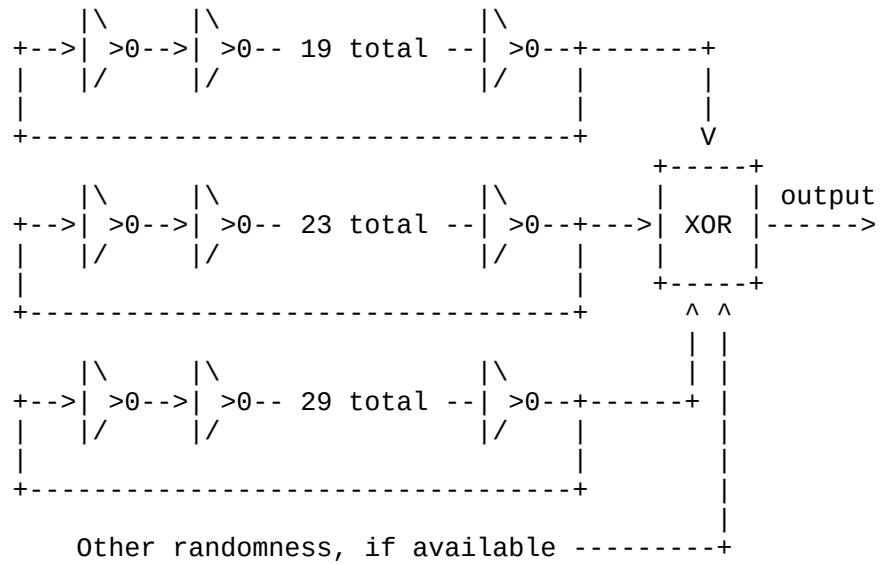
Note: the inclusion of cache memories in disk controllers has little effect on this technique if very short seek times, which represent cache hits, are simply ignored.

3.3. Ring Oscillator Sources

If an integrated circuit is being designed or field-programmed, an odd number of gates can be connected in series to produce a free-running ring oscillator. By sampling a point in the ring at a fixed frequency (for example, one determined by a stable crystal oscillator), some amount of entropy can be extracted due to variations in the free-running oscillator timing. It is possible to increase the rate of entropy by XOR'ing sampled values from a few ring oscillators with relatively prime lengths. It is sometimes recommended that an odd number of rings be used so that, even if the rings somehow become synchronously locked to each other, there will still be sampled bit transitions. Another possible source to sample is the output of a noisy diode.

Sampled bits from such sources will have to be heavily de-skewed, as disk rotation timings must be (see Section 4). An engineering study would be needed to determine the amount of entropy being produced depending on the particular design. In any case, these can be good sources whose cost is a trivial amount of hardware by modern standards.

As an example, IEEE 802.11i suggests the circuit below, with due attention in the design to isolation of the rings from each other and from clocked circuits to avoid undesired synchronization, etc., and with extensive post processing [IEEE_802.11i].



3.4. Problems with Clocks and Serial Numbers

Computer clocks and similar operating system or hardware values, provide significantly fewer real bits of unpredictability than might appear from their specifications.

Tests have been done on clocks on numerous systems, and it was found that their behavior can vary widely and in unexpected ways. One version of an operating system running on one set of hardware may actually provide, say, microsecond resolution in a clock, while a different configuration of the "same" system may always provide the same lower bits and only count in the upper bits at much lower resolution. This means that successive reads of the clock may produce identical values even if enough time has passed that the value "should" change based on the nominal clock resolution. There are also cases where frequently reading a clock can produce artificial sequential values, because of extra code that checks for the clock being unchanged between two reads and increases it by one! Designing portable application code to generate unpredictable numbers based on such system clocks is particularly challenging because the system designer does not always know the properties of the system clock.

Use of a hardware serial number (such as an Ethernet MAC address) may also provide fewer bits of uniqueness than one would guess. Such quantities are usually heavily structured, and subfields may have only a limited range of possible values, or values may be easily guessable based on approximate date of manufacture or other data.

For example, it is likely that a company that manufactures both computers and Ethernet adapters will, at least internally, use its own adapters, which significantly limits the range of built-in addresses.

Problems such as those described above make the production of code to generate unpredictable quantities difficult if the code is to be ported across a variety of computer platforms and systems.

3.5. Timing and Value of External Events

It is possible to measure the timing and content of mouse movement, key strokes, and similar user events. This is a reasonable source of unguessable data, with some qualifications. On some machines, input such as key strokes is buffered. Even though the user's inter-keystroke timing may have sufficient variation and unpredictability, there might not be an easy way to access that variation. Another problem is that no standard method exists for sampling timing details. This makes it hard to use this technique to build standard software intended for distribution to a large range of machines.

The amount of mouse movement and the actual key strokes are usually easier to access than timings, but they may yield less unpredictability because the user may provide highly repetitive input.

Other external events, such as network packet arrival times and lengths, can also be used, but only with great care. In particular, the possibility of manipulation of such network traffic measurements by an adversary and the lack of history at system start-up must be carefully considered. If this input is subject to manipulation, it must not be trusted as a source of entropy.

In principle, almost any external sensor, such as raw radio reception or temperature sensing in appropriately equipped computers, can be used. But in each case, careful consideration must be given to how much this data is subject to adversarial manipulation and to how much entropy it can actually provide.

The above techniques are quite powerful against attackers that have no access to the quantities being measured. For example, these techniques would be powerful against offline attackers who had no access to one's environment and who were trying to crack one's random seed after the fact. In all cases, the more accurately one can measure the timing or value of an external sensor, the more rapidly one can generate bits.

3.6. Non-hardware Sources of Randomness

The best source of input entropy would be a hardware-based random source such as ring oscillators, disk drive timing, thermal noise, or radioactive decay. However, if none of these is available, there are other possibilities. These include system clocks, system or input/output buffers, user/system/hardware/network serial numbers or addresses and timing, and user input. Unfortunately, each of these sources can produce very limited or predictable values under some circumstances.

Some of the sources listed above would be quite strong on multi-user systems, where each user of the system is in essence a source of randomness. However, on a small single-user or embedded system, especially at start-up, it might be possible for an adversary to assemble a similar configuration. This could give the adversary inputs to the mixing process that were well-enough correlated to those used originally to make exhaustive search practical.

The use of multiple random inputs with a strong mixing function is recommended and can overcome weakness in any particular input. The timing and content of requested "random" user keystrokes can yield hundreds of random bits, but conservative assumptions need to be made. For example, one reasonably conservative assumption would be that an inter-keystroke interval provides at most a few bits of randomness, but only when the interval is unique in the sequence of intervals up to that point. A similar assumption would be that a key code provides a few bits of randomness, but only when the code is unique in the sequence. Thus, an interval or key code that duplicated a previous value would be assumed to provide no additional randomness. The results of mixing these timings with typed characters could be further combined with clock values and other inputs.

This strategy may make practical portable code for producing good random numbers for security, even if some of the inputs are very weak on some of the target systems. However, it may still fail against a high-grade attack on small, single-user, or embedded systems, especially if the adversary has ever been able to observe the generation process in the past. A hardware-based random source is still preferable.

4. De-skewing

Is there any specific requirement on the shape of the distribution of quantities gathered for the entropy to produce the random numbers? The good news is that the distribution need not be uniform. All that is needed to bound performance is a conservative estimate of how

non-uniform it is. Simple techniques to de-skew a bit stream are given below, and stronger cryptographic techniques are described in Section 5.2.

4.1. Using Stream Parity to De-Skew

As a simple but not particularly practical example, consider taking a sufficiently long string of bits and mapping the string to "zero" or "one". The mapping will not yield a perfectly uniform distribution, but it can be as close as desired. One mapping that serves the purpose is to take the parity of the string. This has the advantages that it is robust across all degrees of skew up to the estimated maximum skew and that it is trivial to implement in hardware.

The following analysis gives the number of bits that must be sampled:

Suppose that the ratio of ones to zeros is (0.5 + E) to (0.5 - E), where E is between 0 and 0.5 and is a measure of the "eccentricity" of the distribution. Consider the distribution of the parity function of N bit samples. The respective probabilities that the parity will be one or zero will be the sum of the odd or even terms in the binomial expansion of $(p + q)^N$, where p = 0.5 + E, the probability of a one, and q = 0.5 - E, the probability of a zero.

These sums can be computed easily as

$$1/2 * ((p + q)^N + (p - q)^N)$$

and

$$1/2 * ((p + q)^N - (p - q)^N).$$

(Which formula corresponds to the probability that the parity will be 1 depends on whether N is odd or even.)

Since p + q = 1 and p - q = 2E, these expressions reduce to

$$1/2 * [1 + (2E)^N]$$

and

$$1/2 * [1 - (2E)^N].$$

Neither of these will ever be exactly 0.5 unless E is zero, but we can bring them arbitrarily close to 0.5. If we want the probabilities to be within some delta d of 0.5, e.g., then

$$(0.5 + (0.5 * (2E)^N)) < 0.5 + d.$$

Solving for N yields N > log(2d)/log(2E). (Note that 2E is less than 1, so its log is negative. Division by a negative number reverses the sense of an inequality.)

The following table gives the length N of the string that must be sampled for various degrees of skew in order to come within 0.001 of a 50/50 distribution.

Prob(1)	E	N
0.5	0.00	1
0.6	0.10	4
0.7	0.20	7
0.8	0.30	13
0.9	0.40	28
0.95	0.45	59
0.99	0.49	308

The last entry shows that even if the distribution is skewed 99% in favor of ones, the parity of a string of 308 samples will be within 0.001 of a 50/50 distribution. But, as we shall see in section 5.2, there are much stronger techniques that extract more of the available entropy.

4.2. Using Transition Mappings to De-Skew

Another technique, originally due to von Neumann [VON_NEUMANN], is to examine a bit stream as a sequence of non-overlapping pairs. One could then discard any 00 or 11 pairs found, interpret 01 as a 0 and 10 as a 1. Assume that the probability of a 1 is 0.5+E and that the probability of a 0 is 0.5-E, where E is the eccentricity of the source as described in the previous section. Then the probability of each pair is shown in the following table:

pair	probability
00	(0.5 - E)^2 = 0.25 - E + E^2
01	(0.5 - E)*(0.5 + E) = 0.25 - E^2
10	(0.5 + E)*(0.5 - E) = 0.25 - E^2
11	(0.5 + E)^2 = 0.25 + E + E^2

This technique will completely eliminate any bias but requires an indeterminate number of input bits for any particular desired number of output bits. The probability of any particular pair being discarded is 0.5 + 2E^2, so the expected number of input bits to produce X output bits is X/(0.25 - E^2).

This technique assumes that the bits are from a stream where each bit has the same probability of being a 0 or 1 as any other bit in the stream and that bits are uncorrelated, i.e., that the bits come from identical independent distributions. If alternate bits are from two correlated sources, for example, the above analysis breaks down.

The above technique also provides another illustration of how a simple statistical analysis can mislead if one is not always on the lookout for patterns that could be exploited by an adversary. If the algorithm were misread slightly so that overlapping successive bits pairs were used instead of non-overlapping pairs, the statistical analysis given would be the same. However, instead of providing an unbiased, uncorrelated series of random 1s and 0s, it would produce a totally predictable sequence of exactly alternating 1s and 0s.

4.3. Using FFT to De-Skew

When real-world data consists of strongly correlated bits, it may still contain useful amounts of entropy. This entropy can be extracted through various transforms, the most powerful of which are described in section 5.2 below.

Using the Fourier transform of the data or its optimized variant, the FFT, is interesting primarily for theoretical reasons. It can be shown that this technique will discard strong correlations. If adequate data is processed and if remaining correlations decay, spectral lines that approach statistical independence and normally distributed randomness can be produced [BRILLINGER].

4.4. Using Compression to De-Skew

Reversible compression techniques also provide a crude method of de-skewing a skewed bit stream. This follows directly from the definition of reversible compression and the formula in Section 2 for the amount of information in a sequence. Since the compression is reversible, the same amount of information must be present in the shorter output as was present in the longer input. By the Shannon information equation, this is only possible if, on average, the probabilities of the different shorter sequences are more uniformly distributed than were the probabilities of the longer sequences. Therefore, the shorter sequences must be de-skewed relative to the input.

However, many compression techniques add a somewhat predictable preface to their output stream and may insert a similar sequence periodically in their output or otherwise introduce subtle patterns of their own. They should be considered only rough techniques compared to those described in Section 5.2. At a minimum, the beginning of the compressed sequence should be skipped and only later bits should used for applications requiring roughly-random bits.

5. Mixing

What is the best overall strategy for obtaining unguessable random numbers in the absence of a strong, reliable hardware entropy source? It is to obtain input from a number of uncorrelated sources and to mix them with a strong mixing function. Such a function will preserve the entropy present in any of the sources, even if other quantities being combined happen to be fixed or easily guessable (low entropy). This approach may be advisable even with a good hardware source, as hardware can also fail. However, this should be weighed against a possible increase in the chance of overall failure due to added software complexity.

Once one has used good sources, such as some of those listed in Section 3, and mixed them as described in this section, one has a strong seed. This can then be used to produce large quantities of cryptographically strong material as described in Sections 6 and 7.

A strong mixing function is one that combines inputs and produces an output in which each output bit is a different complex non-linear function of all the input bits. On average, changing any input bit will change about half the output bits. But because the relationship is complex and non-linear, no particular output bit is guaranteed to change when any particular input bit is changed.

Consider the problem of converting a stream of bits that is skewed towards 0 or 1 or which has a somewhat predictable pattern to a shorter stream which is more random, as discussed in Section 4. This is simply another case where a strong mixing function is desired, to mix the input bits and produce a smaller number of output bits. The technique given in Section 4.1, using the parity of a number of bits, is simply the result of successively XORing them. This is examined as a trivial mixing function, immediately below. Use of stronger mixing functions to extract more of the randomness in a stream of skewed bits is examined in Section 5.2. See also [NASLUND].

5.1. A Trivial Mixing Function

For expository purposes we describe a trivial example for single bit inputs using the Exclusive Or (XOR) function. This function is equivalent to addition without carry, as show in the table below. This is a degenerate case in which the one output bit always changes for a change in either input bit. But, despite its simplicity, it provides a useful illustration.

input 1	input 2	output
0	0	0
0	1	1
1	0	1
1	1	0

If inputs 1 and 2 are uncorrelated and combined in this fashion, then the output will be an even better (less skewed) random bit than the inputs are. If we assume an "eccentricity" E as defined in Section 4.1 above, then the output eccentricity relates to the input eccentricity as follows:

$$E_{output} = 2 * E_{input\ 1} * E_{input\ 2}$$

Since E is never greater than 1/2, the eccentricity is always improved, except in the case in which at least one input is a totally skewed constant. This is illustrated in the following table, where the top and left side values are the two input eccentricities and the entries are the output eccentricity:

E	0.00	0.10	0.20	0.30	0.40	0.50
0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.10	0.00	0.02	0.04	0.06	0.08	0.10
0.20	0.00	0.04	0.08	0.12	0.16	0.20
0.30	0.00	0.06	0.12	0.18	0.24	0.30
0.40	0.00	0.08	0.16	0.24	0.32	0.40
0.50	0.00	0.10	0.20	0.30	0.40	0.50

However, note that the above calculations assume that the inputs are not correlated. If the inputs were, say, the parity of the number of minutes from midnight on two clocks accurate to a few seconds, then each might appear random if sampled at random intervals much longer

than a minute. Yet if they were both sampled and combined with XOR, the result would be zero most of the time.

5.2. Stronger Mixing Functions

The US Government Advanced Encryption Standard [AES] is an example of a strong mixing function for multiple bit quantities. It takes up to 384 bits of input (128 bits of "data" and 256 bits of "key") and produces 128 bits of output, each of which is dependent on a complex non-linear function of all input bits. Other encryption functions with this characteristic, such as [DES], can also be used by considering them to mix all of their key and data input bits.

Another good family of mixing functions is the "message digest" or hashing functions such as the US Government Secure Hash Standards [SHA*] and the MD4, MD5 [MD4, MD5] series. These functions all take a practically unlimited amount of input and produce a relatively short fixed-length output mixing all the input bits. The MD* series produces 128 bits of output, SHA-1 produces 160 bits, and other SHA functions produce up to 512 bits.

Although the message digest functions are designed for variable amounts of input, AES and other encryption functions can also be used to combine any number of inputs. If 128 bits of output is adequate, the inputs can be packed into a 128-bit data quantity and successive AES "keys", padding with zeros if needed; the quantity is then successively encrypted by the "keys" using AES in Electronic Codebook Mode. Alternatively, the input could be packed into one 128-bit key and multiple data blocks and a CBC-MAC could be calculated [MODES].

More complex mixing should be used if more than 128 bits of output are needed and one wants to employ AES (but note that it is absolutely impossible to get more bits of "randomness" out than are put in). For example, suppose that inputs are packed into three quantities, A, B, and C. One may use AES to encrypt A with B and then with C as keys to produce the first part of the output, then encrypt B with C and then A for more output and, if necessary, encrypt C with A and then B for yet more output. Still more output can be produced by reversing the order of the keys given above. The same can be done with the hash functions, hashing various subsets of the input data or different copies of the input data with different prefixes and/or suffixes to produce multiple outputs.

For an example of using a strong mixing function, reconsider the case of a string of 308 bits, each of which is biased 99% toward zero. The parity technique given in Section 4.1 reduces this to one bit, with only a 1/1000 deviance from being equally likely a zero or one. But, applying the equation for information given in Section 2, this

308-bit skewed sequence contains over 5 bits of information. Thus, hashing it with SHA-1 and taking the bottom 5 bits of the result would yield 5 unbiased random bits and not the single bit given by calculating the parity of the string. Alternatively, for some applications, you could use the entire hash output to retain almost all of the 5+ bits of entropy in a 160-bit quantity.

5.3. Using S-Boxes for Mixing

Many modern block encryption functions, including DES and AES, incorporate modules known as S-Boxes (substitution boxes). These produce a smaller number of outputs from a larger number of inputs through a complex non-linear mixing function that has the effect of concentrating limited entropy from the inputs into the output.

S-Boxes sometimes incorporate bent Boolean functions (functions of an even number of bits producing one output bit with maximum non-linearity). Looking at the output for all input pairs differing in any particular bit position, exactly half the outputs are different. An S-Box in which each output bit is produced by a bent function such that any linear combination of these functions is also a bent function is called a "perfect S-Box".

S-boxes and various repeated applications or cascades of such boxes can be used for mixing [SBOX1, SBOX2].

5.4. Diffie-Hellman as a Mixing Function

Diffie-Hellman exponential key exchange is a technique that yields a shared secret between two parties. It can be computationally infeasible for a third party to determine this secret even if they can observe all the messages between the two communicating parties. This shared secret is a mixture of initial quantities generated by each of the parties [D-H].

If these initial quantities are random and uncorrelated, then the shared secret combines their entropy but, of course, can not produce more randomness than the size of the shared secret generated.

Although this is true if the Diffie-Hellman computation is performed privately, an adversary who can observe either of the public keys and knows the modulus being used need only search through the space of the other secret key in order to be able to calculate the shared secret [D-H]. So, conservatively, it would be best to consider public Diffie-Hellman to produce a quantity whose guessability corresponds to the worse of the two inputs. Because of this and the fact that Diffie-Hellman is computationally intensive, its use as a mixing function is not recommended.

5.5. Using a Mixing Function to Stretch Random Bits

Although it is not necessary for a mixing function to produce the same or fewer output bits than its inputs, mixing bits cannot "stretch" the amount of random unpredictability present in the inputs. Thus, four inputs of 32 bits each, in which there are 12 bits worth of unpredictability (such as 4,096 equally probable values) in each input, cannot produce more than 48 bits worth of unpredictable output. The output can be expanded to hundreds or thousands of bits by, for example, mixing with successive integers, but the clever adversary's search space is still 2^48 possibilities. Furthermore, mixing to fewer bits than are input will tend to strengthen the randomness of the output.

The last table in Section 5.1 shows that mixing a random bit with a constant bit with Exclusive Or will produce a random bit. While this is true, it does not provide a way to "stretch" one random bit into more than one. If, for example, a random bit is mixed with a 0 and then with a 1, this produces a two bit sequence but it will always be either 01 or 10. Since there are only two possible values, there is still only the one bit of original randomness.

5.6. Other Factors in Choosing a Mixing Function

For local use, AES has the advantages that it has been widely tested for flaws, is reasonably efficient in software, and is widely documented and implemented with hardware and software implementations available all over the world including open source code. The SHA* family have had a little less study and tend to require more CPU cycles than AES but there is no reason to believe they are flawed. Both SHA* and MD5 were derived from the earlier MD4 algorithm. They all have source code available [SHA*, MD4, MD5]. Some signs of weakness have been found in MD4 and MD5. In particular, MD4 has only three rounds and there are several independent breaks of the first two or last two rounds. And some collisions have been found in MD5 output.

AES was selected by a robust, public, and international process. It and SHA* have been vouched for by the US National Security Agency (NSA) on the basis of criteria that mostly remain secret, as was DES. While this has been the cause of much speculation and doubt, investigation of DES over the years has indicated that NSA involvement in modifications to its design, which originated with IBM, was primarily to strengthen it. There has been no announcement of a concealed or special weakness being found in DES. It is likely that the NSA modifications to MD4 to produce the SHA algorithms similarly strengthened these algorithms, possibly against threats not yet known in the public cryptographic community.

Where input lengths are unpredictable, hash algorithms are more convenient to use than block encryption algorithms since they are generally designed to accept variable length inputs. Block encryption algorithms generally require an additional padding algorithm to accommodate inputs that are not an even multiple of the block size.

As of the time of this document, the authors know of no patent claims to the basic AES, DES, SHA*, MD4, and MD5 algorithms other than patents for which an irrevocable royalty free license has been granted to the world. There may, of course, be essential patents of which the authors are unaware or patents on implementations or uses or other relevant patents issued or to be issued.

6. Pseudo-random Number Generators

When a seed has sufficient entropy, from input as described in Section 3 and possibly de-skewed and mixed as described in Sections 4 and 5, one can algorithmically extend that seed to produce a large number of cryptographically-strong random quantities. Such algorithms are platform independent and can operate in the same fashion on any computer. For the algorithms to be secure, their input and internal workings must be protected from adversarial observation.

The design of such pseudo-random number generation algorithms, like the design of symmetric encryption algorithms, is not a task for amateurs. Section 6.1 below lists a number of bad ideas that failed algorithms have used. To learn what works, skip Section 6.1 and just read the remainder of this section and Section 7, which describes and references some standard pseudo random number generation algorithms. See Section 7 and Part 3 of [X9.82].

6.1. Some Bad Ideas

The subsections below describe a number of ideas that might seem reasonable but that lead to insecure pseudo-random number generation.

6.1.1. The Fallacy of Complex Manipulation

One approach that may give a misleading appearance of unpredictability is to take a very complex algorithm (or an excellent traditional pseudo-random number generator with good statistical properties) and to calculate a cryptographic key by starting with limited data such as the computer system clock value as the seed. Adversaries who knew roughly when the generator was started would have a relatively small number of seed values to test, as they would know likely values of the system clock. Large numbers of pseudo-

random bits could be generated, but the search space that an adversary would need to check could be quite small.

Thus, very strong or complex manipulation of data will not help if the adversary can learn what the manipulation is and if there is not enough entropy in the starting seed value. They can usually use the limited number of results stemming from a limited number of seed values to defeat security.

Another serious strategic error is to assume that a very complex pseudo-random number generation algorithm will produce strong random numbers, when there has been no theory behind or analysis of the algorithm. There is a excellent example of this fallacy near the beginning of Chapter 3 in [KNUTH], where the author describes a complex algorithm. It was intended that the machine language program corresponding to the algorithm would be so complicated that a person trying to read the code without comments wouldn't know what the program was doing. Unfortunately, actual use of this algorithm showed that it almost immediately converged to a single repeated value in one case and a small cycle of values in another case.

Not only does complex manipulation not help you if you have a limited range of seeds, but blindly-chosen complex manipulation can destroy the entropy in a good seed!

6.1.2. The Fallacy of Selection from a Large Database

Another approach that can give a misleading appearance of unpredictability is to randomly select a quantity from a database and to assume that its strength is related to the total number of bits in the database. For example, typical USENET servers process many megabytes of information per day [USENET_1, USENET_2]. Assume that a random quantity was selected by fetching 32 bytes of data from a random starting point in this data. This does not yield 32*8 = 256 bits worth of unguessability. Even if much of the data is human language that contains no more than 2 or 3 bits of information per byte, it doesn't yield 32*2 = 64 bits of unguessability. For an adversary with access to the same Usenet database, the unguessability rests only on the starting point of the selection. That is perhaps a little over a couple of dozen bits of unguessability.

The same argument applies to selecting sequences from the data on a publicly available CD/DVD recording or any other large public database. If the adversary has access to the same database, this "selection from a large volume of data" step buys little. However, if a selection can be made from data to which the adversary has no access, such as system buffers on an active multi-user system, it may be of help.

6.1.3. Traditional Pseudo-random Sequences

This section talks about traditional sources of deterministic or "pseudo-random" numbers. These typically start with a "seed" quantity and use simple numeric or logical operations to produce a sequence of values. Note that none of the techniques discussed in this section is suitable for cryptographic use. They are presented for general information.

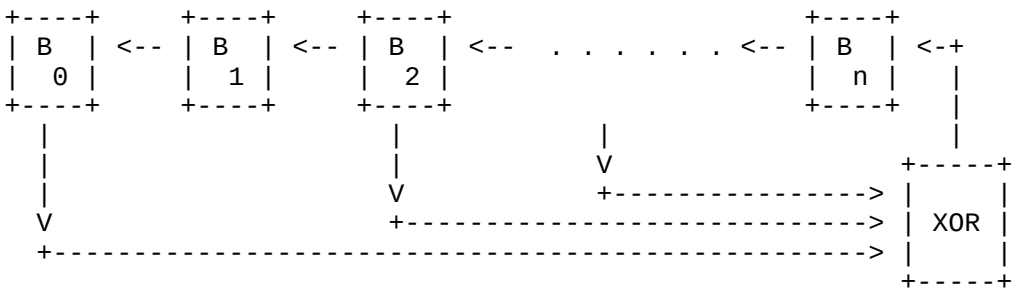
[KNUTH] has a classic exposition on pseudo-random numbers. Applications he mentions are simulations of natural phenomena, sampling, numerical analysis, testing computer programs, decision making, and games. None of these have the same characteristics as the sorts of security uses we are talking about. Only in the last two could there be an adversary trying to find the random quantity. However, in these cases, the adversary normally has only a single chance to use a guessed value. In guessing passwords or attempting to break an encryption scheme, the adversary normally has many, perhaps unlimited, chances at guessing the correct value. Sometimes the adversary can store the message to be broken and repeatedly attack it. Adversaries are also be assumed to be aided by a computer.

For testing the "randomness" of numbers, Knuth suggests a variety of measures, including statistical and spectral. These tests check things like autocorrelation between different parts of a "random" sequence or distribution of its values. But these tests could be met by a constant stored random sequence, such as the "random" sequence printed in the CRC Standard Mathematical Tables [CRC]. Despite meeting all the tests suggested by Knuth, that sequence is unsuitable for cryptographic us, as adversaries must be assumed to have copies of all commonly published "random" sequences and to be able to spot the source and predict future values.

A typical pseudo-random number generation technique is the linear congruence pseudo-random number generator. This technique uses modular arithmetic, where the value numbered N+1 is calculated from the value numbered N by

$$V_{N+1} = (V_N * a + b)(\mathrm{Mod}\ c)$$

The above technique has a strong relationship to linear shift register pseudo-random number generators, which are well understood cryptographically [SHIFT*]. In such generators, bits are introduced at one end of a shift register as the Exclusive Or (binary sum without carry) of bits from selected fixed taps into the register. For example, consider the following:



$$V_{N+1} = ((V_N * 2) + B_0 \ XOR \ B_2 \ldots)(Mod\ 2^n)$$

The quality of traditional pseudo-random number generator algorithms is measured by statistical tests on such sequences. Carefully-chosen values a, b, c, and initial V or carefully-chosen placement of the shift register tap in the above simple process can produce excellent statistics.

These sequences may be adequate in simulations (Monte Carlo experiments) as long as the sequence is orthogonal to the structure of the space being explored. Even there, subtle patterns may cause problems. However, such sequences are clearly bad for use in security applications. They are fully predictable if the initial state is known. Depending on the form of the pseudo-random number generator, the sequence may be determinable from observation of a short portion of the sequence [SCHNEIER, STERN]. For example, with the generators above, one can determine V(n+1) given knowledge of V(n). In fact, it has been shown that with these techniques, even if only one bit of the pseudo-random values are released, the seed can be determined from short sequences.

Not only have linear congruent generators been broken, but techniques are now known for breaking all polynomial congruent generators [KRAWCZYK].

6.2. Cryptographically Strong Sequences

In cases where a series of random quantities must be generated, an adversary may learn some values in the sequence. In general, adversaries should not be able to predict other values from the ones that they know.

The correct technique is to start with a strong random seed, to take cryptographically strong steps from that seed [FERGUSON, SCHNEIER], and not to reveal the complete state of the generator in the sequence elements. If each value in the sequence can be calculated in a fixed

way from the previous value, then when any value is compromised, all future values can be determined. This would be the case, for example, if each value were a constant function of the previously used values, even if the function were a very strong, non-invertible message digest function.

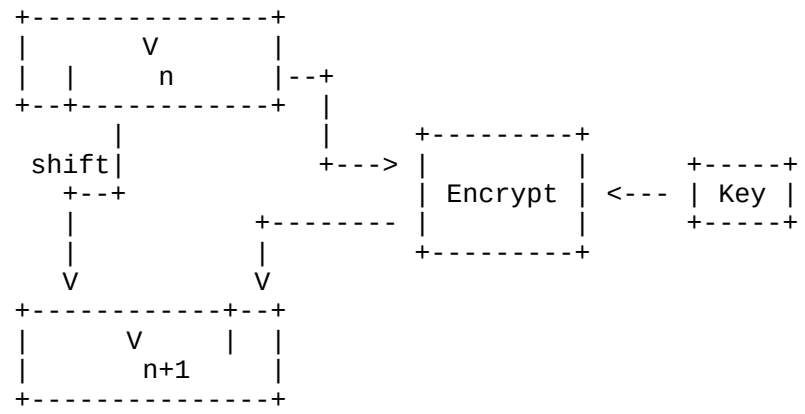
(Note that if a technique for generating a sequence of key values is fast enough, it can trivially be used as the basis for a confidentiality system. If two parties use the same sequence generation technique and start with the same seed material, they will generate identical sequences. These could, for example, be XOR'ed at one end with data being sent to encrypt it, and XOR'ed with this data as received to decrypt it, due to the reversible properties of the XOR operation. This is commonly referred to as a simple stream cipher.)

6.2.1. OFB and CTR Sequences

One way to produce a strong sequence is to take a seed value and hash the quantities produced by concatenating the seed with successive integers, or the like, and then to mask the values obtained so as to limit the amount of generator state available to the adversary.

It may also be possible to use an "encryption" algorithm with a random key and seed value to encrypt successive integers, as in counter (CTR) mode encryption. Alternatively, one can feedback all of the output value from encryption into the value to be encrypted for the next iteration. This is a particular example of output feedback mode (OFB) [MODES].

An example is shown below in which shifting and masking are used to combine part of the output feedback with part of the old input. This type of partial feedback should be avoided for reasons described below.



Note that if a shift of one is used, this is the same as the shift register technique described in Section 6.1.3, but with the all-important difference that the feedback is determined by a complex non-linear function of all bits rather than by a simple linear or polynomial combination of output from a few bit position taps.

Donald W. Davies showed that this sort of shifted partial output feedback significantly weakens an algorithm, compared to feeding all the output bits back as input. In particular, for DES, repeatedly encrypting a full 64-bit quantity will give an expected repeat in about 2^63 iterations. Feeding back anything less than 64 (and more than 0) bits will give an expected repeat in between 2^31 and 2^32 iterations!

To predict values of a sequence from others when the sequence was generated by these techniques is equivalent to breaking the cryptosystem or to inverting the "non-invertible" hashing with only partial information available. The less information revealed in each iteration, the harder it will be for an adversary to predict the sequence. Thus it is best to use only one bit from each value. It has been shown that in some cases this makes it impossible to break a system even when the cryptographic system is invertible and could be broken if all of each generated value were revealed.

6.2.2. The Blum Blum Shub Sequence Generator

Currently the generator which has the strongest public proof of strength is called the Blum Blum Shub generator, named after its inventors [BBS]. It is also very simple and is based on quadratic residues. Its only disadvantage is that it is computationally intensive compared to the traditional techniques given in Section 6.1.3. This is not a major drawback if it is used for moderately-infrequent purposes, such as generating session keys.

Simply choose two large prime numbers (say, p and q) that each gives a remainder of 3 when divided by 4. Let n = p * q. Then choose a random number, x, that is relatively prime to n. The initial seed for the generator and the method for calculating subsequent values are then:

$$s_0 = (x^2)(\mathrm{Mod}\ n)$$

$$s_{i+1} = (s_i^2)(\mathrm{Mod}\ n)$$

Be careful to use only a few bits from the bottom of each s. It is always safe to use only the lowest-order bit. If one uses no more than the:

$$\log_2 (\log_2 (s_i))$$

low-order bits, then predicting any additional bits from a sequence generated in this manner is provably as hard as factoring n. As long as the initial x is secret, n can be made public if desired.

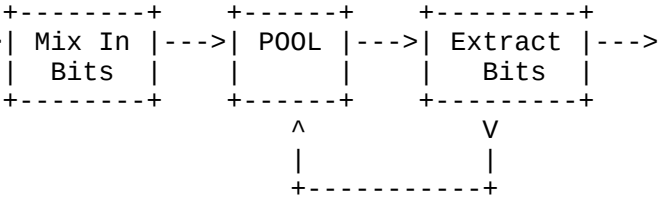
An interesting characteristic of this generator is that any of the s values can be directly calculated. In particular,

$$s_i = (s_0^{((2^i) (\mathrm{Mod} ((p-1)*(q-1))))})(\mathrm{Mod}\ n)$$

This means that in applications where many keys are generated in this fashion, it is not necessary to save them all. Each key can be effectively indexed and recovered from that small index and the initial s and n.

6.3. Entropy Pool Techniques

Many modern pseudo-random number sources, such as those described in Sections 7.1.2 and 7.1.3 utilize the technique of maintaining a "pool" of bits and providing operations for strongly mixing input with some randomness into the pool and extracting pseudo-random bits from the pool. This is illustrated in the figure below.



Bits to be fed into the pool can come from any of the various hardware, environmental, or user input sources discussed above. It is also common to save the state of the pool on system shutdown and to restore it on re-starting, when stable storage is available.

Care must be taken that enough entropy has been added to the pool to support particular output uses desired. See [RSA_BULL1] for similar suggestions.

7. Randomness Generation Examples and Standards

Several public standards and widely deployed examples are now in place for the generation of keys or other cryptographically random quantities. Some, in section 7.1, include an entropy source. Others, described in section 7.2, provide the pseudo-random number strong-sequence generator but assume the input of a random seed or input from a source of entropy.

7.1. Complete Randomness Generators

Three standards are described below. The two older standards use DES, with its 64-bit block and key size limit, but any equally strong or stronger mixing function could be substituted [DES]. The third is a more modern and stronger standard based on SHA-1 [SHA*]. Lastly, the widely deployed modern UNIX and Windows random number generators are described.

7.1.1. US DoD Recommendations for Password Generation

The United States Department of Defense has specific recommendations for password generation [DoD]. It suggests using the US Data Encryption Standard [DES] in Output Feedback Mode [MODES] as follows:

```
Use an initialization vector determined from
     the system clock,
     system ID,
     user ID, and
     date and time;
use a key determined from
     system interrupt registers,
     system status registers, and
     system counters; and,
as plain text, use an external randomly generated 64-bit
quantity such as the ASCII bytes for 8 characters typed
in by a system administrator.
```

The password can then be calculated from the 64 bit "cipher text" generated by DES in 64-bit Output Feedback Mode. As many bits as are needed can be taken from these 64 bits and expanded into a pronounceable word, phrase, or other format if a human being needs to remember the password.

7.1.2. The /dev/random Device

Several versions of the UNIX operating system provide a kernel-resident random number generator. Some of these generators use events captured by the Kernel during normal system operation.

For example, on some versions of Linux, the generator consists of a random pool of 512 bytes represented as 128 words of 4 bytes each. When an event occurs, such as a disk drive interrupt, the time of the event is XOR'ed into the pool, and the pool is stirred via a primitive polynomial of degree 128. The pool itself is treated as a ring buffer, with new data being XOR'ed (after stirring with the polynomial) across the entire pool.

Each call that adds entropy to the pool estimates the amount of likely true entropy the input contains. The pool itself contains a accumulator that estimates the total over all entropy of the pool.

Input events come from several sources, as listed below. Unfortunately, for server machines without human operators, the first and third are not available, and entropy may be added slowly in that case.

1. Keyboard interrupts. The time of the interrupt and the scan code are added to the pool. This in effect adds entropy from the human operator by measuring inter-keystroke arrival times.

2. Disk completion and other interrupts. A system being used by a person will likely have a hard-to-predict pattern of disk

accesses. (But not all disk drivers support capturing this timing information with sufficient accuracy to be useful.)

3. Mouse motion. The timing and mouse position are added in.

When random bytes are required, the pool is hashed with SHA-1 [SHA*] to yield the returned bytes of randomness. If more bytes are required than the output of SHA-1 (20 bytes), then the hashed output is stirred back into the pool and a new hash is performed to obtain the next 20 bytes. As bytes are removed from the pool, the estimate of entropy is correspondingly decremented.

To ensure a reasonably random pool upon system startup, the standard startup and shutdown scripts save the pool to a disk file at shutdown and read this file at system startup.

There are two user-exported interfaces. /dev/random returns bytes from the pool but blocks when the estimated entropy drops to zero. As entropy is added to the pool from events, more data becomes available via /dev/random. Random data obtained from such a /dev/random device is suitable for key generation for long term keys, if enough random bits are in the pool or are added in a reasonable amount of time.

/dev/urandom works like /dev/random; however, it provides data even when the entropy estimate for the random pool drops to zero. This may be adequate for session keys or for other key generation tasks for which blocking to await more random bits is not acceptable. The risk of continuing to take data even when the pool's entropy estimate is small in that past output may be computable from current output, provided that an attacker can reverse SHA-1. Given that SHA-1 is designed to be non-invertible, this is a reasonable risk.

To obtain random numbers under Linux, Solaris, or other UNIX systems equipped with code as described above, all an application has to do is open either /dev/random or /dev/urandom and read the desired number of bytes.

(The Linux Random device was written by Theodore Ts'o. It was based loosely on the random number generator in PGP 2.X and PGP 3.0 (aka PGP 5.0).)

7.1.3. Windows CryptGenRandom

Microsoft's recommendation to users of the widely deployed Windows operating system is generally to use the CryptGenRandom pseudo-random number generation call with the CryptAPI cryptographic service provider. This takes a handle to a cryptographic service provider

library, a pointer to a buffer by which the caller can provide entropy and into which the generated pseudo-randomness is returned, and an indication of how many octets of randomness are desired.

The Windows CryptAPI cryptographic service provider stores a seed state variable with every user. When CryptGenRandom is called, this is combined with any randomness provided in the call and with various system and user data such as the process ID, thread ID, system clock, system time, system counter, memory status, free disk clusters, and hashed user environment block. This data is all fed to SHA-1, and the output is used to seed an RC4 key stream. That key stream is used to produce the pseudo-random data requested and to update the user's seed state variable.

Users of Windows ".NET" will probably find it easier to use the RNGCryptoServiceProvider.GetBytes method interface.

For further information, see [WSC].

7.2. Generators Assuming a Source of Entropy

The pseudo-random number generators described in the following three sections all assume that a seed value with sufficient entropy is provided to them. They then generate a strong sequence (see Section 6.2) from that seed.

7.2.1. X9.82 Pseudo-Random Number Generation

The ANSI X9F1 committee is in the final stages of creating a standard for random number generation covering both true randomness generators and pseudo-random number generators. It includes a number of pseudo-random number generators based on hash functions, one of which will probably be based on HMAC SHA hash constructs [RFC2104]. The draft version of this generator is described below, omitting a number of optional features [X9.82].

In the subsections below, the HMAC hash construct is simply referred to as HMAC but, of course, a particular standard SHA function must be selected in an particular use. Generally speaking, if the strength of the pseudo-random values to be generated is to be N bits, the SHA function chosen must generate N or more bits of output, and a source of at least N bits of input entropy will be required. The same hash function must be used throughout an instantiation of this generator.

7.2.1.1. Notation

In the following sections, the notation give below is used:

hash_length is the output size of the underlying hash function in use.

input_entropy is the input bit string that provides entropy to the generator.

K is a bit string of size hash_length that is part of the state of the generator and is updated at least once each time random bits are generated.

V is a bit string of size hash_length and is part of the state of the generator. It is updated each time hash_length bits of output are generated.

"|" represents concatenation.

7.2.1.2. Initializing the Generator

Set V to all zero bytes, except the low-order bit of each byte is set to one.

Set K to all zero bytes, then set:

```
K = HMAC ( K, V | 0x00 | input_entropy )

V = HMAC ( K, V )

K = HMAC ( K, V | 0x01 | input_entropy )

V = HMAC ( K, V )
```

Note: All SHA algorithms produce an integral number of bytes, so the lengths of K and V will be integral numbers of bytes.

7.2.1.3. Generating Random Bits

When output is called for, simply set:

```
V = HMAC ( K, V )
```

and use the leading bits from V. If more bits are needed than the length of V, set "temp" to a null bit string and then repeatedly perform:

```
V = HMAC ( K, V )
temp = temp | V
```

stopping as soon as temp is equal to or longer than the number of random bits requested. Use the requested number of leading bits from temp. The definition of the algorithm prohibits requesting more than 2^35 bits.

After extracting and saving the pseudo-random output bits as described above, before returning you must also perform two more HMACs as follows:

```
K = HMAC ( K, V | 0x00 )
V = HMAC ( K, V )
```

7.2.2. X9.17 Key Generation

The American National Standards Institute has specified the following method for generating a sequence of keys [X9.17]:

s_0 is the initial 64 bit seed.

g_n is the sequence of generated 64-bit key quantities

k is a random key reserved for generating this key sequence.

t is the time at which a key is generated, to as fine a resolution as is available (up to 64 bits).

DES (K, Q) is the DES encryption of quantity Q with key K.

Then:

$$g_n = DES (k, DES (k, t) XOR s_n)$$

$$s_{n+1} = DES (k, DES (k, t) XOR g_n)$$

If g sub n is to be used as a DES key, then every eighth bit should be adjusted for parity for that use, but the entire 64 bit unmodified g should be used in calculating the next s.

7.2.3. DSS Pseudo-random Number Generation

Appendix 3 of the NIST Digital Signature Standard [DSS] provides a method of producing a sequence of pseudo-random 160 bit quantities for use as private keys or the like. This has been modified by Change Notice 1 [DSS_CN1] to produce the following algorithm for generating general-purpose pseudo-random numbers:

```
t = 0x 67452301 EFCDAB89 98BADCFE 10325476 C3D2E1F0

XKEY  = initial seed
    0

For j = 0 to ...

    XVAL = ( XKEY  + optional user input ) (Mod 2^512)
                j

    X  = G( t, XVAL )
     j

    XKEY   = ( 1 + XKEY  + X  ) (Mod 2^512)
        j+1            j    j
```

The quantities X thus produced are the pseudo-random sequence of 160-bit values. Two functions can be used for "G" above. Each produces a 160-bit value and takes two arguments, a 160-bit value and a 512 bit value.

The first is based on SHA-1 and works by setting the 5 linking variables, denoted H with subscripts in the SHA-1 specification, to the first argument divided into fifths. Then steps (a) through (e) of section 7 of the NIST SHA-1 specification are run over the second argument as if it were a 512-bit data block. The values of the linking variable after those steps are then concatenated to produce the output of G [SHA*].

As an alternative method, NIST also defined an alternate G function based on multiple applications of the DES encryption function [DSS].

8. Examples of Randomness Required

Below are two examples showing rough calculations of randomness needed for security. The first is for moderate security passwords, while the second assumes a need for a very high-security cryptographic key.

In addition, [ORMAN] and [RSA_BULL13] provide information on the public key lengths that should be used for exchanging symmetric keys.

8.1. Password Generation

Assume that user passwords change once a year and that it is desired that the probability that an adversary could guess the password for a particular account be less than one in a thousand. Further assume that sending a password to the system is the only way to try a password. Then the crucial question is how often an adversary can try possibilities. Assume that delays have been introduced into a system so that an adversary can make at most one password try every six seconds. That's 600 per hour, or about 15,000 per day, or about 5,000,000 tries in a year. Assuming any sort of monitoring, it is unlikely that someone could actually try continuously for a year. Even if log files are only checked monthly, 500,000 tries is more plausible before the attack is noticed and steps are taken to change passwords and make it harder to try more passwords.

To have a one-in-a-thousand chance of guessing the password in 500,000 tries implies a universe of at least 500,000,000 passwords, or about 2^29. Thus, 29 bits of randomness are needed. This can probably be achieved by using the US DoD-recommended inputs for password generation, as it has 8 inputs that probably average over 5 bits of randomness each (see section 7.1). Using a list of 1,000 words, the password could be expressed as a three-word phrase (1,000,000,000 possibilities). By using case-insensitive letters and digits, six characters would suffice ((26+10)^6 = 2,176,782,336 possibilities).

For a higher-security password, the number of bits required goes up. To decrease the probability by 1,000 requires increasing the universe of passwords by the same factor, which adds about 10 bits. Thus, to have only a one in a million chance of a password being guessed under the above scenario would require 39 bits of randomness and a password that was a four-word phrase from a 1,000 word list, or eight letters/digits. To go to a one-in-10^9 chance, 49 bits of randomness are needed, implying a five-word phrase or a ten-letter/digit password.

In a real system, of course, there are other factors. For example, the larger and harder to remember passwords are, the more likely users will bed to write them down, resulting in an additional risk of compromise.

8.2. A Very High Security Cryptographic Key

Assume that a very high security key is needed for symmetric encryption/decryption between two parties. Assume also that an adversary can observe communications and knows the algorithm being used. Within the field of random possibilities, the adversary can try key values in hopes of finding the one in use. Assume further that brute force trial of keys is the best the adversary can do.

8.2.1. Effort per Key Trial

How much effort will it take to try each key? For very high-security applications, it is best to assume a low value of effort. Even if it would clearly take tens of thousands of computer cycles or more to try a single key, there may be some pattern that enables huge blocks of key values to be tested with much less effort per key. Thus, it is probably best to assume no more than a couple of hundred cycles per key. (There is no clear lower bound on this, as computers operate in parallel on a number of bits and a poor encryption algorithm could allow many keys or even groups of keys to be tested in parallel. However, we need to assume some value and can hope that a reasonably strong algorithm has been chosen for our hypothetical high-security task.)

If the adversary can command a highly parallel processor or a large network of work stations, 10^11 cycles per second is probably a minimum assumption today. Looking forward a few years, there should be at least an order of magnitude improvement. Thus, it is reasonable to assume that 10^10 keys could be checked per second, or 3.6*10^12 per hour or 6*10^14 per week, or 2.4*10^15 per month. This implies a need for a minimum of 63 bits of randomness in keys, to be sure that they cannot be found in a month. Even then it is possible that, a few years from now, a highly determined and resourceful adversary could break the key in 2 weeks; on average, they need try only half the keys.

These questions are considered in detail in "Minimal Key Lengths for Symmetric Ciphers to Provide Adequate Commercial Security: A Report by an Ad Hoc Group of Cryptographers and Computer Scientists" [KeyStudy] that was sponsored by the Business Software Alliance. It concluded that a reasonable key length in 1995 for very high security is in the range of 75 to 90 bits and, since the cost of cryptography does not vary much with the key size, it recommends 90 bits. To update these recommendations, just add 2/3 of a bit per year for Moore’s law [MOORE]. This translates to a determination, in the year 2004, a reasonable key length is in the 81- to 96-bit range. In fact, today, it is increasingly common to use keys longer than 96

bits, such as 128-bit (or longer) keys with AES and keys with effective lengths of 112-bits with triple-DES.

8.2.2. Meet-in-the-Middle Attacks

If chosen or known plain text and the resulting encrypted text are available, a "meet-in-the-middle" attack is possible if the structure of the encryption algorithm allows it. (In a known plain text attack, the adversary knows all or part (possibly some standard header or trailer fields) of the messages being encrypted. In a chosen plain text attack, the adversary can force some chosen plain text to be encrypted, possibly by "leaking" an exciting text that is sent by the adversary over an encrypted channel because the text is so interesting.

The following is an oversimplified explanation of the meet-in-the-middle attack: the adversary can half-encrypt the known or chosen plain text with all possible first half-keys, sort the output, and then half-decrypt the encoded text with all the second half-keys. If a match is found, the full key can be assembled from the halves and used to decrypt other parts of the message or other messages. At its best, this type of attack can halve the exponent of the work required by the adversary while adding a very large but roughly constant factor of effort. Thus, if this attack can be mounted, a doubling of the amount of randomness in the very strong key to a minimum of 192 bits (96*2) is required for the year 2004, based on the [KeyStudy] analysis.

This amount of randomness is well beyond the limit of that in the inputs recommended by the US DoD for password generation and could require user-typing timing, hardware random number generation, or other sources of randomness.

The meet-in-the-middle attack assumes that the cryptographic algorithm can be decomposed in this way. Hopefully no modern algorithm has this weakness, but there may be cases where we are not sure of that or even of what algorithm a key will be used with. Even if a basic algorithm is not subject to a meet-in-the-middle attack, an attempt to produce a stronger algorithm by applying the basic algorithm twice (or two different algorithms sequentially) with different keys will gain less added security than would be expected. Such a composite algorithm would be subject to a meet-in-the-middle attack.

Enormous resources may be required to mount a meet-in-the-middle attack, but they are probably within the range of the national security services of a major nation. Essentially all nations spy on other nations' traffic.

8.2.3. Other Considerations

[KeyStudy] also considers the possibilities of special-purpose code-breaking hardware and having an adequate safety margin.

Note that key length calculations such as those above are controversial and depend on various assumptions about the cryptographic algorithms in use. In some cases, a professional with a deep knowledge of algorithm-breaking techniques and of the strength of the algorithm in use could be satisfied with less than half of the 192 bit key size derived above.

For further examples of conservative design principles, see [FERGUSON].

9. Conclusion

Generation of unguessable "random" secret quantities for security use is an essential but difficult task.

Hardware techniques for producing the needed entropy would be relatively simple. In particular, the volume and quality would not need to be high, and existing computer hardware, such as audio input or disk drives, can be used.

Widely-available computational techniques can process low-quality random quantities from multiple sources, or a larger quantity of such low-quality input from one source, to produce a smaller quantity of higher-quality keying material. In the absence of hardware sources of randomness, a variety of user and software sources can frequently, with care, be used instead. However, most modern systems already have hardware, such as disk drives or audio input, that could be used to produce high-quality randomness.

Once a sufficient quantity of high-quality seed key material (a couple of hundred bits) is available, computational techniques are available to produce cryptographically-strong sequences of computationally-unpredictable quantities from this seed material.

10. Security Considerations

The entirety of this document concerns techniques and recommendations for generating unguessable "random" quantities for use as passwords, cryptographic keys, initialization vectors, sequence numbers, and similar security applications.

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Appendix A: Changes from RFC 1750

1. Additional acknowledgements have been added.

2. Insertion of section 5.3 on mixing with S-boxes.

3. Addition of section 3.3 on Ring Oscillator randomness sources.

4. Addition of AES and the members of the SHA series producing more than 160 bits. Use of AES has been emphasized and the use of DES de-emphasized.

5. Addition of section 6.3 on entropy pool techniques.

6. Addition of section 7.2.3 on the pseudo-random number generation techniques given in FIPS 186-2 (with Change Notice 1), 7.2.1 on those given in X9.82, section 7.1.2 on the random number generation techniques of the /dev/random device in Linux and other UNIX systems, and section 7.1.3 on random number generation techniques in the Windows operating system.

7. Addition of references to the "Minimal Key Lengths for Symmetric Ciphers to Provide Adequate Commercial Security" study published in January 1996 [KeyStudy] and to [RFC1948].

8. Added caveats to using Diffie-Hellman as a mixing function and, because of those caveats and its computationally intensive nature, recommend against its use.

9. Addition of references to the X9.82 effort and the [TURBID] and [NASLUND] papers.

10. Addition of discussion of min-entropy and Renyi entropy and references to the [LUBY] book.

11. Major restructuring, minor wording changes, and a variety of reference updates.

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Part 2 - Non-Deterministic Random Bit Generators
Part 3 - Deterministic Random Bit Generators

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